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Single Antenna Attitude Algorithm for Nonuniform Antenna Gain Patterns

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The objective of this research is to investigate a new method for improving the accuracy of single antenna attitude systems based on global positioning system signal strength measurements. To achieve this objective, a predictive global positioning system signal strength model is proposed and developed. The model consists of a precisely measured 3-D receiving antenna gain pattern (as fitted to the spacecraft), distance dependent path loss, effects of the ionosphere, global positioning system satellite transmitting antenna gain, and variation in global positioning system satellite transmission power. Furthermore, an algorithm is developed to provide an estimate of single-axis attitude solution based on this predictive global positioning system signal strength model. The performance of this new algorithm is evaluated and compared with two other single antenna attitude approaches using ground data and flight data. Results show that the new single antenna attitude algorithm is capable of providing an attitude accuracy of 10 deg rms for static terrestrial platforms with a zenith pointing antenna. Furthermore, 15 deg rms from the FedSat satellite negative velocity pointing antenna configuration has been achieved.

Nomenclature

b^L	=	estimate of the antenna boresight, deg
G_r	=	receiving antenna gain, dB
$G_{r(\text{estimated})}$	=	estimated receiving antenna gain, dB
$G_{r(\text{measured})}$	=	measured receiving antenna gain, dB
G_t	=	global positioning system transmitting antenna gain, dB
L_I	=	loss in analog to digital conversion, dB
L_{Nf}	=	global positioning system receiver noise figure, dB
P_t	=	global positioning system transmitting power, dBW
PL	=	path losses, dB
p	=	total number of global positioning system signal observation
R	=	range, m
T_{sky}	=	sky noise, K
VL	=	vacuum loss of global positioning system signal strength, dB
α	=	off-boresight angle, deg
σ	=	standard deviation

I. Introduction

OVER the last decade, substantial research has been directed toward the use of global positioning system (GPS) signal strength measurements for attitude estimation [1–9]. Initially, this work focused on the single antenna configuration and is capable of providing single-axis attitude to within 15 deg accuracy. More recently, research has extended the work to multiple antenna configurations for solving single-axis limitation and improving system performance. These systems have been primarily developed for space applications, where the attitude solution can be useful in the role of emergency backup or for integer ambiguity resolution in a multiple antenna baselines attitude system [1].

Compared with multiple antenna approaches, multiple antenna baselines, and multiple antenna signal strength measurement, the main advantage of the single antenna configuration is the simple and low-cost hardware requirement. In addition, this is an algorithmic approach, and can be implemented on any system where a GPS receiver that measures signal strength is present. Thus, for small satellite missions where performance vs weight vs cost tradeoffs are common, the single antenna approach is an attractive option and often investigated.

The achievable accuracy of single antenna attitude systems has been reported as 15 deg rms [1,3,9]. Given that this approach is strongly influenced by errors in measured signal strength, it was hypothesized that a predictive GPS signal strength model composed of comprehensive GPS signal transmission link budgets, evaluated for each GPS satellite at each epoch, could provide an improvement to system performance.

An overview of the paper is as follows: In Sec. II, a brief overview of single antenna attitude systems is presented. In particular, potential error sources that have caused poor attitude solution performance are introduced. Section III presents system performance analysis in a simulated environment where the error source was minimized. Results obtained from this experiment provide an indication of the theoretical best performance achievable. Section IV presents an investigation into the GPS signal transmission model in an attempt to minimize the signal strength modeling error and improve system performance. GPS signal transmission link budget parameters such as receiving antenna gain, distance dependent path loss, the effects of ionosphere, GPS satellite transmitting antenna gain, and varying GPS satellite transmitting power are considered. Section V presents a new single antenna attitude system based on the GPS signal transmission model developed. In particular, the importance of the use of a three dimensional receiving antenna gain pattern in the algorithm is addressed. Section VI investigates the performance of the new attitude system by comparing with both the Duncan approach and the Axelrad approach. Finally, conclusions and recommendations are made in Sec. VII.

II. Single Antenna Attitude Systems Overview

The primary GPS observable used in most single antenna attitude systems is the signal strength measurement [1,5–7,9]. This approach

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relies on the assumption that the GPS receiving antenna gain reduces monotonically from the boresight vector to 90 deg off-boresight. The off-boresight angle is illustrated in Fig. 1. In addition, it is sometimes assumed that the azimuthal variation of the receiver antenna gain is very small and can be ignored. Through the measurement of all GPS satellite signal strengths and the known geometry of the tracked satellites, the orientation of the antenna boresight vector, with respect to a reference coordinate system, can be estimated.

There are two published approaches under investigation. The Duncan approach [3], was developed by the NASA Jet Propulsion Laboratory (JPL) in June 1998. The estimate of the antenna boresight direction (the single-axis solution) is solved as the weighted average of the line-of-sight (LOS) vectors from the GPS receiver antenna to each GPS satellite being tracked. The weights are assigned based on the measured C/N_0 value such that the GPS satellites with high C/N_0 measurement will have the higher weight applied to their LOS vector. It was shown that when six to eight GPS satellites are being tracked, the estimated boresight differs from the truth by no more than 15 deg [3].

The second approach, developed by Axelrad [1] involves the development of a model of the GPS receiver antenna received signal strength as a function of off-boresight angle. The procedure involves the collection of signal strength measurement with a known attitude reference that can be performed before the launch of the satellite or in orbit. Once the signal strength to off-boresight angle mapping model has been created, the antenna boresight vector can be estimated through the minimization of a cost function [1]. Results have shown that the accuracy of the Axelrad approach varies between 3.2 to 11.9 deg rms for space-borne data [1] and an accuracy of 10 to 15 deg for ground data [9].

Researchers have indicated that the performance of these systems depends on several factors such as signal strength measurement, number of GPS satellite signals available, and the geometry of the tracked GPS satellites [1–9]. For an optimal system performance, signal strength measurement must be compensated for the signal strength gains and losses that occurred during transmission. These parameters include GPS transmitting power, GPS transmitting antenna gain, distant dependent path loss, receiving antenna gain, and receiver dependent errors. In addition, signal strength measurement is also influenced by an external environment such as multipath effects and ionospheric scintillation.

Investigation into the Duncan and Axelrad approaches has indicated that one of the main causes of coarse accuracy is inaccurate modeling of the GPS signal transmission link budget parameters. For example, both the Duncan and the Axelrad approaches were developed based on the assumption that receiving antenna gain varies as a function of the off-boresight angle and ignores the receiving antenna gain variation in the azimuth direction (azimuthal symmetry). However, as will be shown in Sec. IV, this is not always an appropriate assumption. The receiving antenna gain pattern can be easily distorted by its proximity to the surrounding surface. In addition, knowledge of the GPS satellite transmitting antenna gain pattern is very limited. The actual gain versus elevation response of satellite flight antennas mounted on the spacecraft was not measured before launch, only the manufacturer-supplied reference gain pattern was available [10]. Furthermore, the gain pattern of Block IIR satellites is known to have a slightly different gain pattern compared with earlier Block II/IIA satellites [11]. However, very limited information was published about this issue and was not considered in the Duncan and the Axelrad approaches. Finally, the actual transmitted power of the GPS satellite is known to vary with time. However, neither approach has compensated for the differences in transmitted power level in their algorithm.

III. Single Antenna Attitude System Best Performance Analysis

Both the Duncan and the Axelrad approaches have shown that their accuracy is greatly dependent on the GPS signal strength measurement. Any noise in the signal strength measurement will result in a decrease of attitude determination performance. Potential error sources include receiver noises, multipath, scintillation, and poor assumptions for the antenna gain patterns. To understand the impact of signal strength on the attitude solution performance, a best performance analysis was conducted through the use of a global navigation satellite system (GNSS) constellation simulator.

The experimental setup consisted of a WelNavigate GS-720 GPS constellation simulator, and a Novatel 3151R GPS receiver with multipath elimination technology (MET). During the test, the multipath feature of the simulator was disabled and a receiving antenna gain profile created. Because of simulator limitations, a stairstep antenna gain profile was created, such that for every 5 deg decrease in elevation angle, an extra 1 dB attenuation will be imposed on the signal. It is recognized that the resulting linear antenna gain pattern has a relatively narrow beam width, but for a single antenna attitude experiment, it represents the best case scenario with a linear relationship between elevation angle and antenna gain.

Data were collected over a 2 h simulation and applied to the Axelrad approach. This approach was chosen because it is more dependent on signal strength measurement than the Duncan approach [3]. First, the data were analyzed to investigate noise on the GPS receiver signal strength measurement. Noise is determined by removing antenna gain attenuation from signal strength measurements and then normalizing to zero about the mean value. It should be noted that the resulting noise estimate is due to both the receiver and the simulator; however, it still provides a good indication of the GPS receiver noise level. Figure 2 shows the noise calculated for a pseudorandom number (PRN) 23 satellite with $\sigma = 0.3896$ dB/Hz. This noise level ultimately limits the accuracy of signal strength based methods.

The attitude estimation results of the experiment are shown in Fig. 3. The left graph a) shows the minimum, average, and maximum signal strength (C/N_0) for all satellites as a function of elevation angle. As can be seen, the received C/N_0 increases with increasing elevation angle. Apart from several outliers, the difference between the maximum and the minimum signal strength is approximately 3 dB. Because data were collected in a multipath free environment and the receiving antenna gain pattern carefully defined, signal strength variations are due to noise in the GPS receiver and the simulator. Figure 3b shows the antenna boresight estimation error with the Axelrad approach. The mean and standard deviation of the pointing error are 0.8 and 5.1 deg, respectively. Compared with the 15 deg accuracy previously achieved by these researchers [1,3,9], the

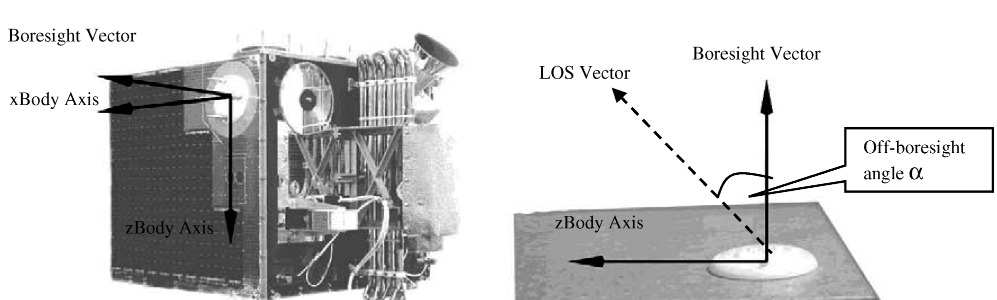

Fig. 1 Antenna boresight definition.

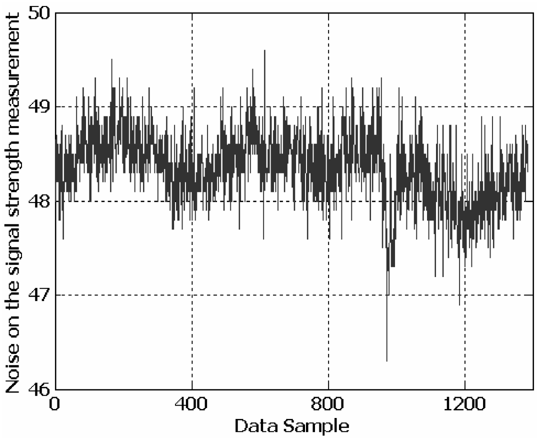

Fig. 2 Signal strength measurement noise for PRN23 satellite in simulated environment.

simulation environment result provides an improvement in accuracy by eliminating multiple error sources.

IV. GPS Signal Transmission Gain/Loss Parameters

As shown from the results of the experiment in Sec. II, the coarse accuracy of a single antenna attitude system was a result of the inability to correctly model GPS signal transmission gain and loss parameters. To overcome this problem, parameters that could potentially influence GPS signal strength must be modeled. A detailed discussion of each of these parameters is provided in this section. The following GPS signal model is developed similar to work presented by Moreau [10]. Figure 4 shows a basic GPS signal transmission path.

Based on the GPS signal transmission path model in Fig. 4, the received power P_r or GPS signal power at the output of the receiving antenna, can be written as

$$P_r = P_t + G_t + PL + G_r \tag{1}$$

The relationship between carrier-to-noise spectral density C/N_0 and P_r can be expressed as

$$C/N_0 = P_r - 10^*\log_{10} T_{\text{sky}} - (-228.6) + L_{\text{Nf}} + L_I \tag{2}$$

where T_{sky} is the noise introduced from the environment (mostly the sky), −228.6 is the Boltzmann constant (dBW/Hz-K), L_{Nf} is the noise figure of the GPS receiver, and L_I is the loss in analog to digital (A/D) conversion. Thus, by substituting Eqs. (1) and (2), C/N_0 can be expressed as

$$C/N_0 = P_t + G_t + PL + G_r - 10^*\log_{10} T_{\text{sky}} + 228.6 + L_{\text{Nf}} + L_I \tag{3}$$

Finally, assuming that T_{sky}, L_{Nf}, and L_I stays constant for a given hardware configuration, constant terms in Eq. (3) can be grouped together to form a signal offset c. Thus, C/N_0 measured by the GPS receiver can be expressed as

$$C/N_0 = P_t + G_t + PL + G_r + c \tag{4}$$

Equation (4) shows that the C/N_0 measurements consist of four varying components: GPS transmitting power, GPS transmitting antenna gain, path losses, and receiving antenna gain. Issues related to the modeling of each of four components will be discussed in the following subsections:

A. Receiving Antenna Gain

Receiving antenna gain patterns employed by previous researchers are based on two assumptions [1,3,9]. First, the receiving antenna gain is assumed to be at the highest along the antenna boresight vector and decreasing down to 90 deg off-boresight. Second, gain variation in the azimuthal direction is assumed to be very small and is sometimes ignored. However, these two assumptions are not always correct. Depending on the antenna mounting position, the receiving antenna gain pattern could be distorted by the signal reflected off the mounting surface.

To examine the distortion of the receiving antenna gain pattern due to reflected signal, a replicate of the FedSat microsatellite was tested in an anechoic chamber. It is a galvanized tin cube of the same dimension of FedSat (50 × 50 × 50 cm), referred to as TinSat. Although the structural materials of TinSat are not the same as used in FedSat, it is assumed that general gain pattern variations will be similar. This test platform is chosen to allow for the use of FedSat mission data for system performance analysis. A detailed description

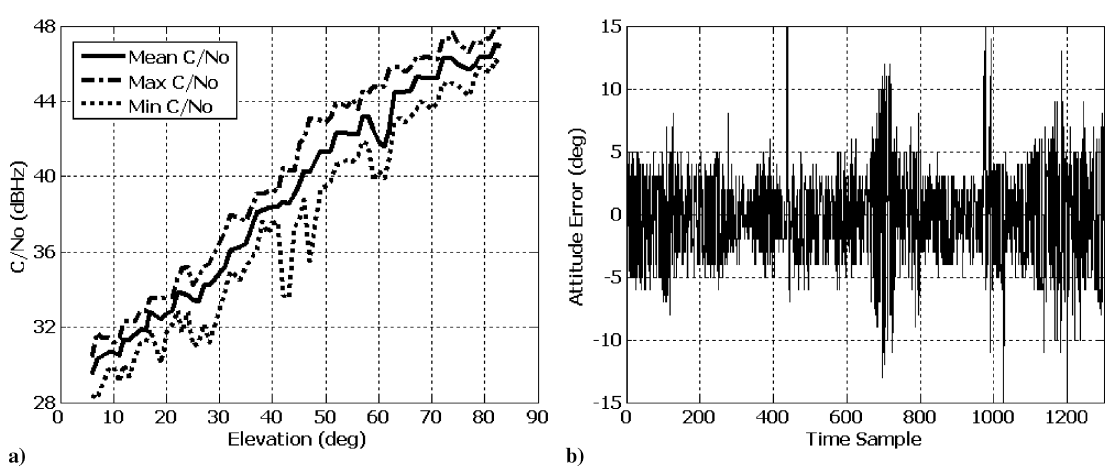

Fig. 3 Simulation results: a) error in signal strength measurement; b) attitude estimation error.

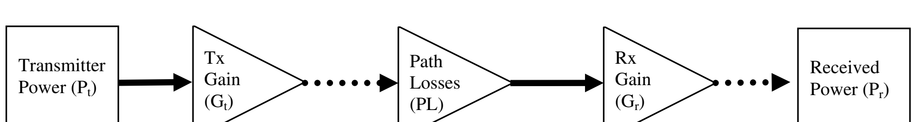

Fig. 4 GPS signal transmission path model.

of test setup and results was documented by Kellar [12], but is described briefly here.

Antenna gain pattern tests were conducted in an anechoic chamber room covered with RF energy absorbers to reduce multipath effects. The antenna used is a dual-band $L1/L2$ passive GPS antenna (Sensor System S67-1575-14). As shown in Fig. 5, the manufacturer's published antenna gain pattern [13] agrees very well with the assumptions made in the Duncan approach and the Axelrad approach.

The Sensor System antenna was mounted on the top right hand corner of the TinSat, shown as the white circle in Fig. 6. A "Scientific Atlanta 1783" microwave receiver was used to acquire the gain, and the phase measurement at 0.5 deg interval. The process was repeated through the four antenna plane slices as defined in Fig. 6.

Figure 7 shows gain along the antenna plane slice, 225 to 45 deg, as defined in Fig. 6. Note that 180 deg shown in Fig. 7 is defined as the antenna boresight vector.

Compared with the manufacturer's published data, the antenna gain test result has been severely distorted. Consider the circle labeled as "I" in Fig. 7, elevation angle of 30 deg, a large increase in the gain value has been observed. The gain at this point is even higher than the gain along the boresight vector of the antenna. This is a result of signals reflected off the mounting surface.

The receiving antenna gain as a function of azimuth and off-boresight angle was created in Matlab as shown in Fig. 8. Because the antenna test performed only on four antenna slices, in between points were interpolated with piecewise cubic Hermite interpolating polynomial function (PCHIP). A lookup table was created for every 0.5 deg interval range from azimuth angle 0 to 360 deg and off-boresight angle 0 to 130 deg.

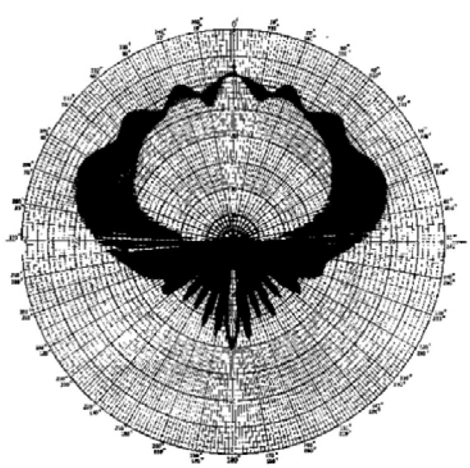

Fig. 5 Published antenna gain pattern for Sensor Systems S67-1575-14 [13].

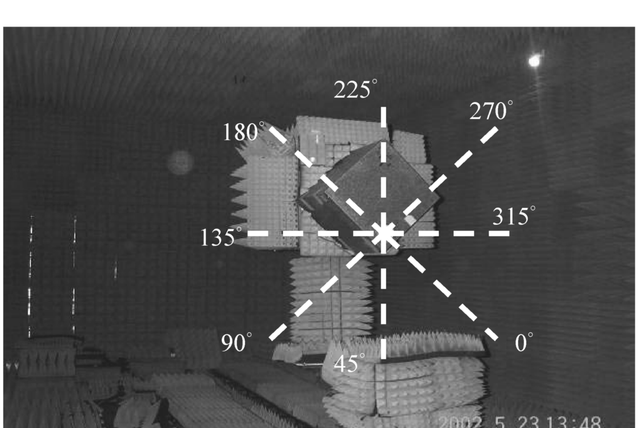

Fig. 6 Definition of TinSat antenna plane slice.

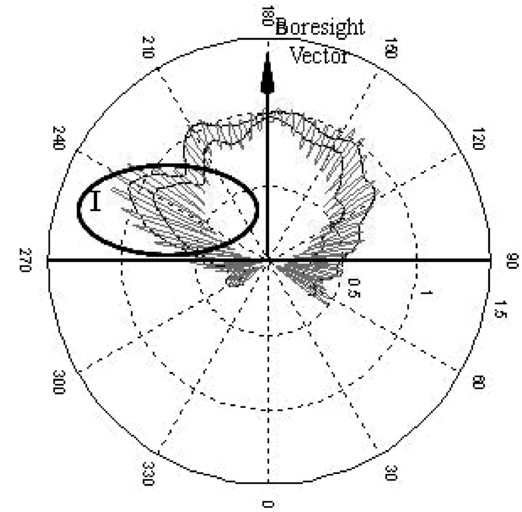

Fig. 7 L1, left to right, 225 to 45 deg slice.

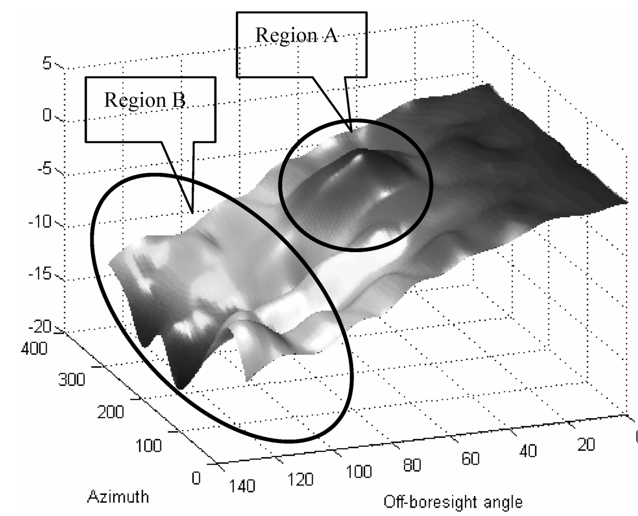

Fig. 8 PCHIP interpolation of the FedSat GPS 3-D antenna gain pattern.

Figure 8 is a three dimensional representation of the Sensor System antenna gain pattern when mounted on one corner of the TinSat. The antenna gain pattern is heavily distorted in region A, located between an azimuth angle of 220 to 270 deg and off-boresight angle of 50 to 70 deg. This distortion corresponds to the distorted region highlighted in Fig. 7, and is caused by the nonuniform ground plane. Region B in Fig. 8 also shows a heavily distorted gain pattern. As this region is beyond a 90 deg off-boresight angle, it means that some of the signals in this region were tracked by the receiver by penetrating around the FedSat surface. As a result, large modeling errors are seen in this region.

B. Path Losses

The amount of loss in GPS signal strength is dependent on both distance of travel and type of transmitting medium. As the GPS signal traveling through space, the signal could encounter two different mediums: vacuum and atmosphere. These two mediums each have different transmission losses associated with them. In addition, the effects of ionospheric scintillation and multipath on signal strength will be discussed in the following:

1. Propagation Through Vacuum

Transmission in a vacuum is primary related to the distance traveled (distance dependent path loss) [1]. With the knowledge of distance between the GPS satellite and the receiving antenna, actual

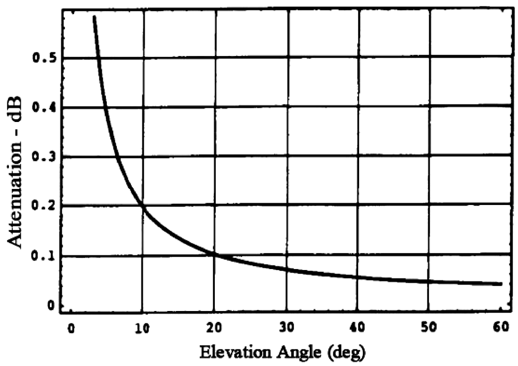

Fig. 9 Atmospheric attenuation [14].

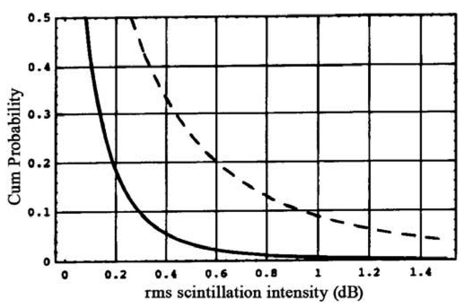

Fig. 10 Cumulative probability for the amplitude scintillation [14].

range, vacuum loss can be calculated as

$$VL(\text{dB}) = 20\log_{10}R \tag{5}$$

2. *Propagation Through Atmosphere*

Atmospheric attenuation losses, including ionosphere and troposphere, in the GPS signal frequency range are primarily due to the presence of oxygen [14]. The model for atmospheric attenuation due to oxygen versus elevation in degrees is shown in Fig. 9. The typical signal attenuation at Earth's surface is 0.035 dB at the zenith and increases by a factor of 10 at low elevation [14]. In addition, the effects of water vapor, rain, and nitrogen attenuation in this frequency are less than 0.5 dB at 5 deg elevation angle [14].

3. *Ionospheric Scintillation*

As the GPS signal traveling through the atmosphere, it can be influenced by the irregularities in the electron content in the ionosphere region. These irregularities can cause amplitude and phase fluctuations in the GPS signal, which is referred to as ionospheric scintillation [15]. Researchers have shown that the amplitude fading at GPS L1 frequency may exceed 20 dB and last for several hours [16]. Ionospheric scintillations are mostly common near the equator after sunset, but can also occur at high latitude during either day or night.

Unfortunately, a model for predicting signal strength loss due to scintillation cannot be accomplished. Fu [16] states that variation in signal strength is not a linear function of electron density [16] and is characterized by probability distributions. The Nakagame-*m* distribution (solid curve in Fig. 10) is used for weak/moderate scintillation, and the Rayleigh distribution (dashed curve in Fig. 10) is used for strong scintillation [16].

4. *Multipath Effect*

GPS signal strength could also be influenced by the multipath effect caused by signal reflections from the surrounding terrain. Multipath errors have more effect on GPS signals received at low elevation angles (grazing incidence) and reduce significantly at higher elevation angles.

The prediction of fluctuation in GPS signal strength due to multipath is very complex and cannot be easily modeled. Thus, it is ideal to eliminate GPS signals that have been most affected by multipath. This can be accomplished by employing an antenna mask, such that any signal received by the receiving antenna with an elevation angle less than 10 deg is eliminated from the calculation. In addition, strategies such as code minus carrier can be employed to estimate the effect of multipath. Signals affected by reflectors that are higher than 10 deg in the field of view can be detected and eliminated.

C. GPS Transmitting Satellite

As of May 2005, the GPS constellation consists of 28 GPS BLOCK II/IIA/IIR satellites. The first two series of the operational satellites, BLOCK II and BLOCK IIA, were developed by Rockwell International Space System Division with an antenna array design composed of 12 helical elements mounted on the Earth-facing satellite panel [11]. The newer GPS BLOCK IIR satellites that began service in 1997 are operational replenishment satellites developed by Lockheed Martin. Compared with earlier BLOCK II/IIA satellites, modifications in the BLOCK IIR antenna array design were made to the ratio of inner and outer radii and the RF power fed ratio [11].

Modeling of the GPS transmitting antenna gain pattern for the BLOCK II/IIA/IIR satellite has been done by several researchers [1,5,9,10]. One of the models developed by Moreau [10] is shown in Fig. 11. Note that the antenna gain of BLOCK IIR satellites is slightly lower than that of BLOCK II/IIA satellites. This is due to the modification in the antenna array design as mentioned. In addition, the GPS transmitting antenna gain pattern is assumed to be azimuthal symmetry.

Although GPS transmitting antenna gain patterns are available, several modeling error sources have been identified. One is the receiving antenna gain pattern used during the experiment. The gain patterns used in the experiments were either from the manufacturer's published data or using onboard calibrated antenna gain model. Both receiving antenna gain pattern modeling approaches are not accurate enough and the error could translate to uncertainties in the GPS transmitting antenna gain pattern developed.

GPS satellite transmitting power is another factor that needs to be taken into consideration. It is often assumed that transmitting power is constant and equal for all GPS satellites [1,10]. However, this is not

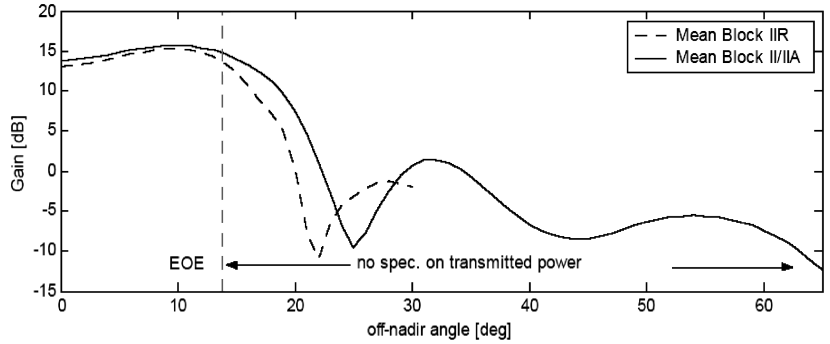

Fig. 11 Mean gain pattern for GPS Block II/IIA/IIR satellites [10].

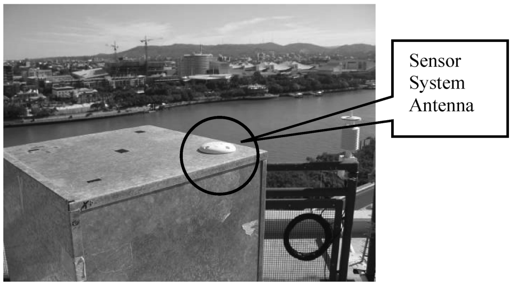

Fig. 12 Test environment setup.

quite true. As indicated in previous studies, transmitting power often exceeds the specified level because it is expected to degrade with time. Differences in the transmitting power level could be as much as 6 dB. Unfortunately, neither the Duncan approach nor the Axelrad approach compensate for this difference in their algorithms.

To solve for these problems, an experiment was set up to reconstruct GPS BLOCK II/IIA/IIR satellite antenna gain patterns and determine the transmitting power differences between GPS satellites. The experiment was conducted on the roof of a 13 story high building as shown in Fig. 12. The Sensor System antenna was mounted on the upward facing panel of TinSat where spirit level was used to ensure alignment of the antenna boresight vector with the gravity vector. A Novatel 3151R GPS receiver with multipath elimination technology (MET) was used to record parameters such as C/N_0 and the receiving antenna position. The GPS satellite position was calculated from ephemeris, and the receiving antenna attitude was measured using spirit level and a compass. As the setup is the same as the platform in the anechoic chamber room test, the accurately measured three dimensional antenna gain pattern could be to obtain more accurate GPS transmitting antenna gain models.

Because there are two unknowns to be solved, it is first assumed that the transmitting antenna gain models presented in Fig. 11 are accurate enough and are used to solve for GPS transmitting power differences. Rearranging Eq. (4), transmitting power can be expressed as

$$P_t + c = C/N_0 - G_r + PL - G_t \tag{6}$$

The transmitting power for each individual GPS satellite can then be calculated using Eq. (6), where parameters are calculated as follows:

1) C/N_0 is measured by the receiver.

2) G_r is determined from the LOS vector, the receiver attitude, and the receiving antenna gain pattern.

3) PL is determined from the GPS satellite position and the receiver position.

4) G_t is determined from the LOS vector, and the transmitting antenna gain model.

5) The mean GPS transmitting power for each GPS satellite is shown in Fig. 13. Note that the value shown in the figure includes a constant term that cannot be separated. Using the lowest value as a reference (PRN 14), differences in the transmitting power between satellites can be then determined.

Figure 13 shows that the PRN 16 satellite has the highest $P_t + c$ of 170.3 dB and is approximately 2.22 dB higher than the lowest value recorded by the PRN 14 satellite. Although the difference in transmitting power is not as large as 6 dB claimed by previous researchers, variation of 2.22 dB is still a large error and has to be modeled.

With the knowledge of $(P_t + c)$ for each GPS satellite, the transmitting antenna gain pattern can be reconstructed. Rearranging Eq. (6):

$$G_t = C/N_0 - G_r + PL - (P_t + c) \tag{7}$$

Models for GPS antenna gain pattern were constructed using GPS signals with an off-boresight angle of less than 70 deg. This mask angle is imposed to remove the multipath affected signal. Antenna gain models are created using best fit 4th order polynomial function.

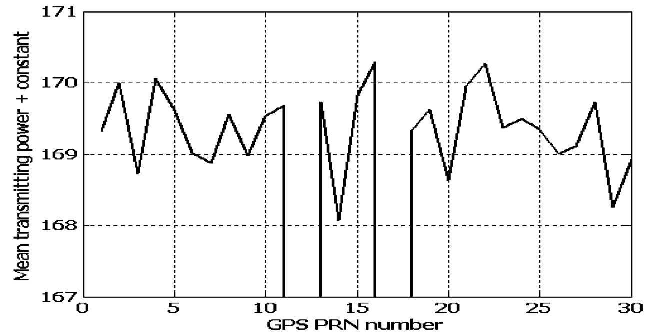

Fig. 13 Mean transmitting power plus a constant term for all GPS satellites.

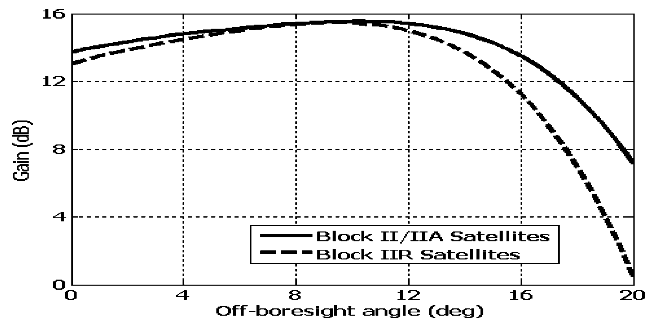

Fig. 14 Antenna gain patterns for GPS Block II/IIA and Block IIR satellites.

Figure 14 shows the reconstructed GPS transmitting antenna gain pattern for BLOCK II/IIA/IIR satellites. In general, the shape and amplitude of the result agree well with the published patterns shown in Fig. 11. Because a precisely measured three dimensional receiving antenna gain pattern was used for the reconstruction work, Fig. 14 is considered to be a better antenna gain model for GPS satellites.

V. Methodologies of the New Single Antenna Attitude Algorithm

Single antenna attitude system performance, especially the Axelrad approach, is strongly dependent on signal strength measurements. Any uncertainty on these measurements reduces the accuracy of the attitude estimate. A potential solution to the problem is to improve the modeling of parameters that affect GPS signal strength during transmission. As presented in Sec. IV, models for four of these parameters are created. These include GPS transmitting power, GPS transmitting antenna gain, path losses, and receiving antenna gain. Other parameters, such as ionospheric scintillation and multipath, are not considered because of their complexity.

During the investigation, it was discovered that implementing a three dimensional receiving antenna gain model could not be done on the Duncan approach and the Axelrad approach. As both algorithms have assumed that receiving antenna gain is a function of off-boresight angle only, the additional gain information in the azimuthal direction cannot be incorporated into the algorithm. Thus, a new single antenna attitude algorithm is developed.

As defined by Wahba, the general form of the attitude estimation problem is to find a proper orthogonal matrix C that defines a transformation from reference frame A to reference frame B. The optimal attitude solution is determined by minimizing the cost function shown in Eq. (8).

$$J(^BC^A) = \frac{1}{2}\sum_{i=1}^{p} a_i |V_i^B - {}^BC^A V_i^A|^2 \tag{8}$$

where a_i is the weight associated with each observation, V_i^A is the unit vector expressed in reference frame A, and V_i^B is the measurement of the same unit vector expressed in reference frame B.

This cost function is modified such that the cost value is determined based on receiving antenna gain values derived from the transmission model presented in Sec. IV.

First, the measured receiving antenna gain can be determined for any given GPS signal strength measurement based on the GPS transmission model. Rearranging Eq. (4), $G_{r(\text{measured})}$ can be written as follows:

$$G_{r(\text{measured})} = C/N_0 - (P_t + c) - G_t - PL \tag{9}$$

C/N_0 is obtained from the output of the GPS receiver. $(P_t + \text{c})$ is determined from Fig. 13 with respect to the PRN of the GPS signal observation. G_t is derived using GPS transmitting antenna gain models presented in Fig. 14 and the positions of the GPS satellite and the receiving antenna. Finally, the PL term is determined using Eq. (6), which is based upon the distance between the positions of GPS satellites and the receiving antenna.

On the other hand, it is possible to estimate the receiving antenna gain of the same GPS signal observation based on the three dimensional receiving antenna gain model and LOS vectors. From an initial b^L, $G_{r(\text{estimated})}$ can be determined by transforming the LOS vector to the satellite's body frame. Finally, rotating the estimated antenna boresight through the space, an optimal estimate of attitude can be found by minimizing the cost function defined as follows:

$$J(b^L) = \frac{1}{2}\sum_{i=1}^{p} |G_{ri(\text{measured})} - G_{ri(\text{estimated})}|^2 \tag{10}$$

VI. Result Analysis

To compare and evaluate results achieved by the new single antenna attitude algorithm (the Wang approach), two algorithms (the Duncan approach and the Axelrad approach) were developed and used for benchmarking. Performance analyses of these three single antenna attitude systems are done using two data sets. The first data set is the actual FedSat flight data (from 30/12/2002 to 28/02/2003). The reference FedSat attitude information is obtained from the onboard attitude control system where the accuracy is within ± 6 deg of the truth for the first few minutes after it is turned on. The second data set is collected in a static terrestrial environment (May 2005) as shown earlier in Fig. 12. The Sensor System antenna was mounted on TinSat in a zenith pointing configuration.

A. FedSat Flight Data

Figure 15 shows the result of attitude estimation using the Duncan approach where the attitude error is defined as the angle between the estimated boresight vector and the reference attitude of FedSat. As shown in Fig. 15, the mean attitude error in the Duncan approach is 26.85 deg and the rms error is approximately 30 deg. This result is not too surprising, because the Duncan approach relies more on the geometry of the tracked satellites. The negative velocity pointing FedSat antenna results in uneven distribution of tracked GPS satellites because part of the antenna view is blocked by the Earth, causing degraded performance.

Attitude estimation results based on the the Axelrad approach are shown in Figs. 16 and 17.

Figure 16a shows the variation in observed C/N_0 for receiving antenna off-boresight angles with respect to the reference frame for

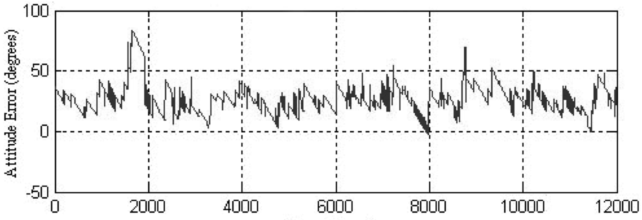

Fig. 15 FedSat attitude estimation error using the Duncan approach.

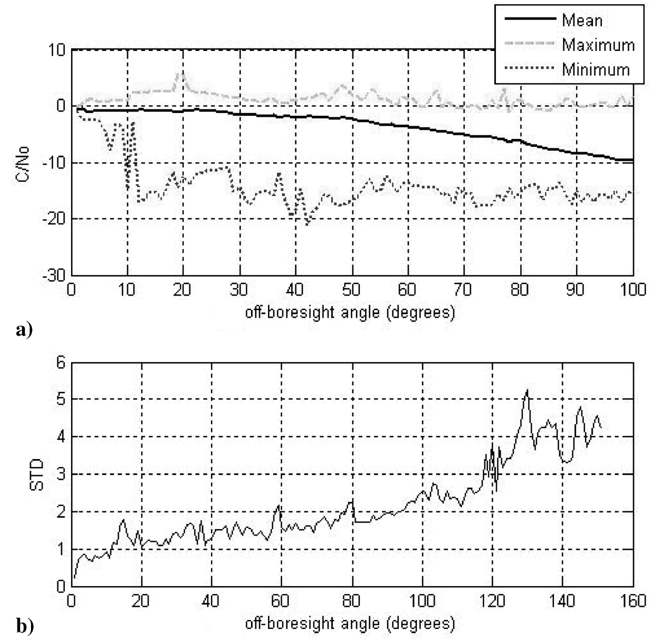

Fig. 16 Signal strength to elevation angle calibration results.

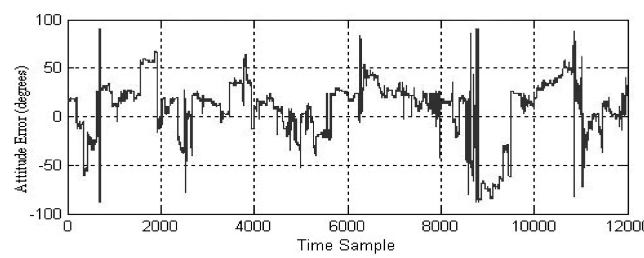

Fig. 17 FedSat attitude estimation error using the Axelrad approach.

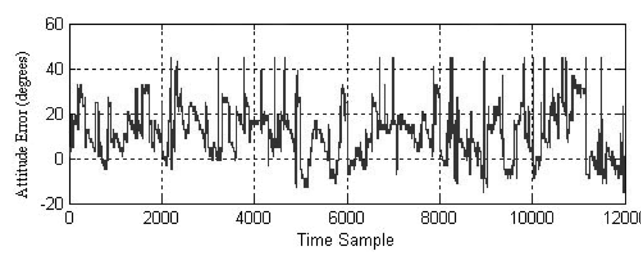

Fig. 18 FedSat attitude estimation error using the Wang approach.

all measurements made, at all azimuths. The maximum, mean and minimum observed C/N_0 are shown in the plot. Figure 16b shows the standard deviation error of the observed C/N_0. Errors in the mapping function were gradually increased as the receiving antenna off-boresight angle increased. However, as the receiving off-boresight angles went beyond 90 deg, errors in modeling raised sharply, reflecting the large azimuthal error shown in region B, Fig. 8. Figure 17 shows the attitude estimation error when using the Axelrad approach. The mean error is approximately 9.3 deg with rms error of 31.1 deg.

Attitude estimation results based on the Wang approach are shown in Fig. 18. As shown from the figure, the attitude estimation result is much better than the performance achieved by the Duncan approach and the Axelrad approach. The Wang approach provides a mean accuracy of 5.66 deg and rms error of 15.24 deg.

In addition, the performance of single antenna attitude systems (Duncan, Axelrad, Wang) are analyzed with respect to the number of GPS satellites being tracked and the results are shown in Table 1, Figs. 19 and 20.

Table 1 System performance with respect to the number of GPS satellites being tracked by the receiver

	Data samples	Duncan, mean error	Duncan, rms error	Axelrad, mean error	Axelrad, rms error	Wang, mean error	Wang, rms error
All samples	12,000	26.85	29.39	9.23	31.11	5.66	15.24
Sat# = 4	813	30.02	32.65	−4.27	38.89	1.50	18.82
Sat# = 5	2022	35.83	39.60	8.34	41.20	5.92	15.72
Sat# = 6	3623	27.65	28.88	5.33	27.97	7.23	12.77
Sat# = 7	3745	23.50	25.03	13.02	26.20	7.24	12.18

The performance of the Duncan and the Axelrad approach achieved in this experiment has been degraded when compared with the results achieved previously [1,3,9]. This is caused by the FedSat antenna configuration in the negative velocity direction and the nonuniform ground plane. On the other hand, the new attitude system based on 3-D receiving antenna gain pattern is less subject to the effects of the negative velocity pointing antenna configuration. Analyzed results also indicate that the performance of the system is dependent on the number of GPS satellites being tracked by the receiver.

B. Static Terrestrial Environment

In addition, three algorithms are analyzed using static terrestrial data, where the data were collected using an upward pointing antenna. Results are shown in the following figures:

Figure 21 shows the result of attitude estimation using the Duncan algorithm for the static terrestrial setup. Compared with results achieved for FedSat flight data, slight improvements in attitude estimation results have been achieved, mean error of 18.8 deg and rms error of 19.8 deg. Figure 22 is the attitude estimation results for the Axelrad approach. Results are slightly better than the Duncan approach with a mean error of 13.4 deg and rms error of 14.3 deg. As shown in Fig. 23, the Wang approach achieved the best result with a mean attitude estimation error of 8.66 deg and rms error of 9.8 deg.

Finally, the system performance of three single antenna attitude system investigated (Duncan, Axelrad, and Wang) are summarized in Table 2. As clearly shown in the table, the Wang approach has achieved the best attitude estimation result in both environments. With the help of GPS signal transmission model development, the

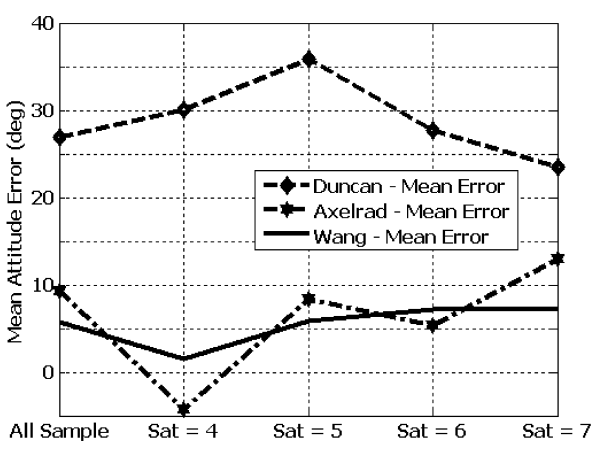

Fig. 19 Mean attitude error.

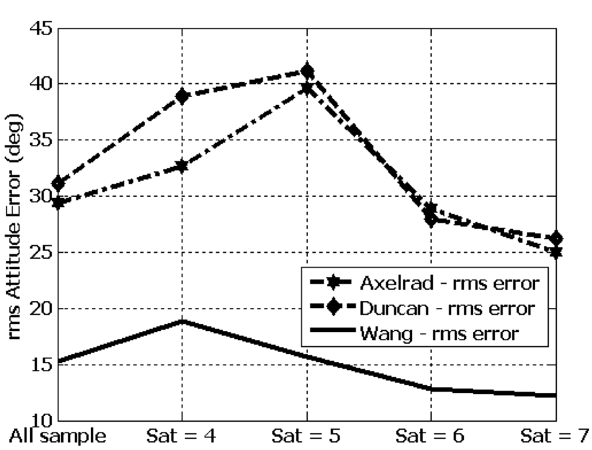

Fig. 20 20 rms attitude error.

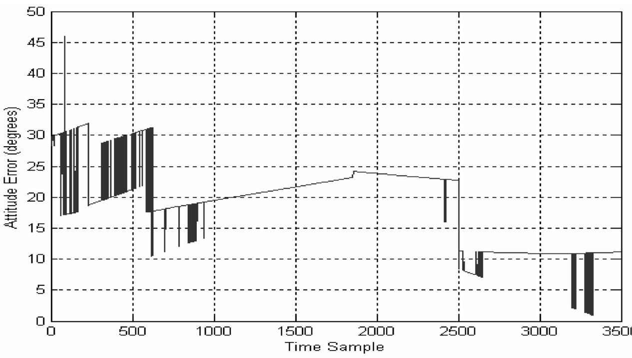

Fig. 21 Static terrestrial attitude estimation error using the Duncan approach.

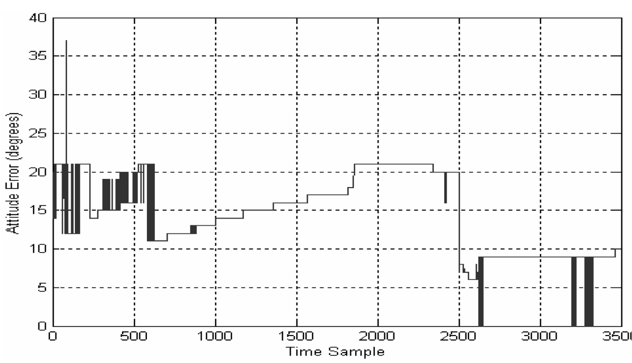

Fig. 22 Static terrestrial attitude estimation error using the Axelrad approach.

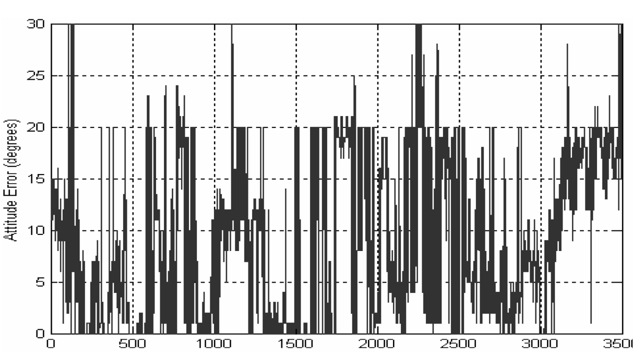

Fig. 23 Static terrestrial attitude estimation error using the Wang approach.

Table 2 Summary of system performances

	Duncan approach	Axelrad approach	Wang approach
FedSat flight data: rms error, deg	29.4	31.1	15.2
Static terrestrial data: rms error, deg	19.8	14.3	9.8

Wang approach is capable of delivering an accuracy of 15 deg rms for space mission and an accuracy of 10 deg for the static terrestrial setup.

VII. Conclusions

Three single antenna attitude systems were implemented and evaluated. Results from actual FedSat flight data have shown a degraded performance from the Duncan and the Axelrad algorithms when compared with results achieved previously by researchers [1,3]. This is due to the negative velocity pointing antenna configuration and the nonuniform ground plane of FedSat. On the other hand, the Wang approach has achieved improved attitude estimation results by incorporating the precisely measured three dimensional receiving antenna gain pattern and the GPS signal transmission model.

On the other hand, results from the static terrestrial data set have shown similar results. Slightly degraded performances have been shown for the Duncan approach and the Axelrad approach due to nonuniform receiving antenna gain pattern. The result for the Wang approach improves to 10 deg compared with 14 deg achieved by the Axelrad approach and 20 deg achieved by the Duncan approach.

Although the Wang approach can achieve an accuracy of only 15 deg rms for FedSat flight data and 10 deg rms for static terrestrial data, several improvements can be made to enhance performance. First, improvements can be made to the modeling of GPS transmitting gain patterns. As discussed in [10], transmitting antenna gain is affected by mechanical antenna alignment errors, the noon-turn maneuver, and the effect of the position of the solar array. At various GPS satellite transmitting theta angles (angles between the nadir vector and the LOS vector), the transmitting gain value can vary as much as 2 dB due to these factors. Thus, it is recommended to obtain an accurate spherical gain pattern of the GPS Block II/IIA/IIR satellite. Furthermore, the first of the new modernized GPS satellites was deployed and declared operational on 16 December 2005. The modernized series GPS IRR-M offers enhanced features such as a new antenna panel to provide increased signal power and a second signal for civilian users. It is expected that these new features will affect the GPS transmitting antenna gain pattern and the transmitting power level. Further study needs to be conducted to determine the impact of these changes.

Another potential performance improvement can be made by incorporating both GNSS satellites and future Galileo satellites into the algorithm. As the performances of single antenna attitude systems are also dependent on the number of tracked satellites, the increase in the number of signals is expected to improve attitude estimation performance.

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